

Report on the Generalized Tanh Method Extended to a Variable-Coefficient Korteweg-de Vries Equation

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We briefly report that the generalized tanh method can be extended from the situation with coefficient constants to that with coefficient functions. Soliton-typed solutions for a variable-coefficient Korteweg-de Vries equation are thus found. Similar work can be done for the generalized variable-coefficient Kadomtsev-Petviashvili equations.

The variable-coefficient Korteweg-de Vries (vcKdV) equations are able to realistically model various physical situations, as seen, e.g., in [1–4].

A generalized tanh method has newly been proposed and applied to several constant-coefficient nonlinear evolution equations [5, 6]. Hereby, we will extend this method to directly solve for a vcKdV equation ([4] and references therein):

$$u_t = h_1(t) (u_{xxx} + 6u u_x) + 4h_2(t) u_x - h_0(t) (2u + x u_x), \quad (1)$$

where all $h_j(t)$'s are arbitrary functions. We assume that certain soliton-typed solutions of (1) are of the form

$$u(x, t) = \sum_{j=0}^N A_j(t) \cdot \tanh^j [\mathcal{F}(t)x + \mathcal{G}(t)], \quad (2)$$

where the $A_j(t)$'s, $\mathcal{F}(t)$ and $\mathcal{G}(t)$ are differentiable functions with $\mathcal{F}(t) \neq 0$ and $A_N(t) \neq 0$, while N is determined via the leading-order analysis as $N=2$. We then substitute Expression 2 into (1) and equate to zero the coefficients of like powers of x and $\tanh(\mathcal{F}x + \mathcal{G})$, so that after computerized symbolic computation we obtain the soliton-typed solutions

$$u(x, t) = \left\{ \beta - 2\alpha^2 + 2\alpha^2 \cdot \operatorname{sech}^2 [\alpha x e^{-\int h_0(t) dt} + 2\alpha(3\beta - 4\alpha^2) \int h_1(t) e^{-3\int h_0(t) dt} dt + 4\alpha \int h_2(t) e^{-\int h_0(t) dt} dt] \right\} \cdot e^{-2\int h_0(t) dt}, \quad (3)$$

where $\alpha \neq 0$ and β are a couple of constants. In [1], the same solutions as (3) were obtained via the inverse scattering. In comparison, the inverse scattering is a well-established, powerful tool, while the technique presented in this note is both concise and straightforward.

Similar work has been done for a generalized variable-coefficient Kadomtsev-Petviashvili equation [7]. We conclude that the generalized tanh method can be successfully extended from the situation with coefficient constants to that with coefficient functions.

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